4.2

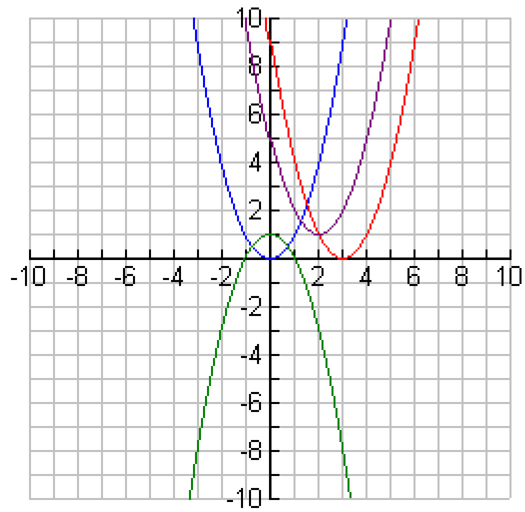
Graphical Transformations

$y = x^2$

$y = (x-3)^2$

$y = 1 - x^2$

$y = x^2 - 4x + 5$



Transformations – functions that map real numbers to real numbers

Rigid Transformations – leave the size and shape of a graph unchanged (horizontal and vertical translations, reflections)

Non-rigid transformations – distort shape of graph (horizontal and vertical stretches and shrinks)

Vertical Translations:

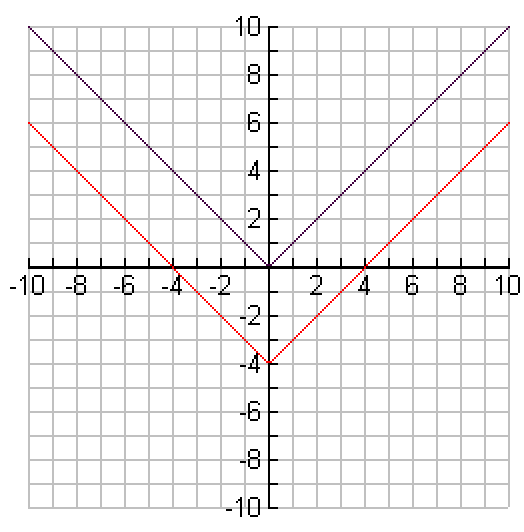
Vertical translation shifts graph up or down.

$y = f(x) + c$ up by c units.

$y = f(x) - c$ down by c units

Ex. $y = |x|$

$y = |x| - 4$



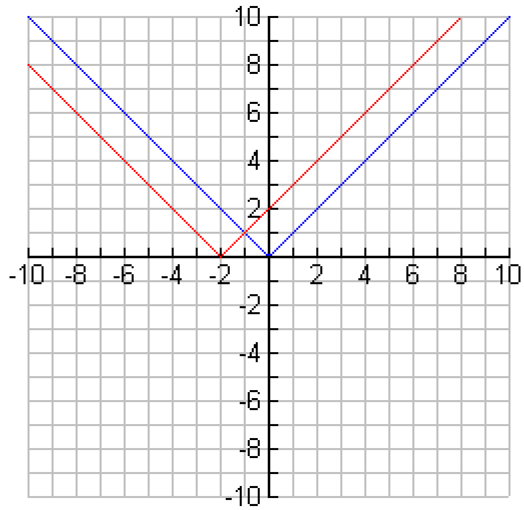
Horizontal Translations:

Horizontal translation shifts graph left or right.

$y = f(x - c)$ to right by c units. $y = f(x + c)$ to left by c units.

$y = |x|$

$y = |x + 2|$



Reflections:

Points (x, y) and $(x, -y)$ are **reflections of each other across the *x*-axis**.

Across the x-axis: $y = f(x) \quad \rightarrow \quad y = -f(x)$

Points (x, y) and $(-x, y)$ are **reflections of each other across the *y*-axis**.

Across the y-axis: $y = f(x) \quad \rightarrow \quad y = f(-x)$

Finding Equations of Reflections:

Find an equation for the reflection of $f(x) = \dfrac{3x - 7}{x^2 + 2}$ across each axis.

Reflection across x-axis: $y = -f(x)$

$$-f(x) = -\left(\frac{3x - 7}{x^2 + 2}\right) = \frac{-3x + 7}{x^2 + 2}$$

Reflections across y – axis: *y = f(-x)*

$$f(-x) = -\left(\frac{3(-x)-7}{(-x)^2+2}\right) = \frac{-3x-7}{x^2+2}$$

Stretches and Shrinks

Let *c* be a positive real number. Then the following transformations result in stretches or shrinks of the graph of *y* = *f*(*x*).

Horizontal Stretches or Shrinks

y = f ($\frac{x}{c}$) a stretch by a factor of *c* if *c* >1

a shrink by a factor of *c* if 0 < *c* < 1

Vertical Stretches or Shrinks

y = c • f(x) a stretch by a factor of *c* if *c* > 1

a shrink by a factor of *c* if 0 < *c* < 1

Examples:

Transform the given function by a) a **vertical stretch** by a factor of 3, and b) a **horizontal shrink** by a factor of 1/2.

$$f(x) = |x+4|$$

a) $3f(x) = 3|x+4|$

b) $f(\frac{x}{1/2}) = f(2x) = |2x+4|$

Sequence of transformations:

1) Reflections, 2) Stretches and shrinks and 3) shifts

Describe a basic graph and a sequence of transformations that can be used to produce a graph of the given function.

$y = -2\sqrt{x+3}$

Starting with $y = \sqrt{x}$, reflect across the x-axis, vertical stretch by factor of 2, and shift left 3.

A new graph is obtained from the series of transformations on the given graph; write the equation for the new graph.

$y = x^2$; a vertical stretch by a factor of 4, then a shift right 6 units.

$y = 4(x-6)^2$

Even function – for all x in the domain of *f: if f*(-x) = *f*(x), symmetric about the y-axis.

$(x, y) \to (-x, y)$

Odd function – for all x in the domain of *f: if f*(-x) = -*f*(x), symmetric about the origin.

$(x, y) \to (-x, -y)$

Examples:

Determine whether the given function is even, odd or neither.

a) $f(x) = x^2 + 4$

Step 1: Find *f(-x)* and if *f(-x) = f(x)*, then it is even.

$f(-x) = (-x)^2 + 4 = x^2 + 4 = f(x)$, so the function is even. We are done.

Step 2: (If necessary)

If not even then check to see if it is odd, *f(-x) = -f(x).*

Try $f(x) = x^2 - 2x - 4$

Step 1:

$f(-x) = (-x)^2 - 2(-x) - 4 = x^2 + 2x - 4 \neq f(x)$, not even.

Step 2:

We know from step 1 that $f(-x) = x^2 + 2x - 4$ so now we find *–f(x).*

$-f(x) = -(x^2 - 2x - 4) = -x^2 + 2x + 4 \neq f(-x)$, so the function is neither even or odd.

Shifty WS. ??